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Axisymmetric Excited Integral Equation Using Moment Method for Plane Circular disk

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***Abstract*—Calculating current distribution is a common problem in antenna. It significantly affects the operation of almost all type of antenna. There have been many different methods suggested to derive the equation for impedance and current distribution so far. In this paper, integral equation using numerical technique for plane circular surface is investigated. An expression for the current distribution (Moment method is employed), is derived. This is the first review in this topic and we believe that it help to give new direction of calculating impedance and current distribution methods especially for parabolic reflector, which have been so far proposed.**

Index Terms—Charge carrier density, Integral equations, Moment methods.

I. Introduction

THE impedance of an antenna depends on many factors like its operating frequency, geometry, method of excitation, and proximity to the surrounding objects. Due to geometries, only few antennas have been investigated theoretically. Impedance of an antenna at a point is defined as the ratio of the electric to the magnetic fields at that point; alternatively at a pair of terminals, it is defined as the ratio of the voltage to the current across those terminals also[1]. There are few methods that can be used to calculate the current distribution of an antenna.

(1) Boundary-value method
(2) Transmission line method
(3) Poynting vector method

Most basic approach is the boundary-value method. The solution to this is obtained by enforcing the boundary conditions (Tangential electric-field components vanish at the conducting surface). Impedance of an antenna can also be found using an integral equation with a numerical technique solution, which is referred to as the integral equation method of moments[2] which cite the solution for the induced current in the form of an integral equation. This method includes electromagnetic problems which is analytically simple, but requires large computation. In this paper the integral equation method, with a Moment Method numerical solution, will be introduced and used to calculate the impedance and current distribution of circular plate. This approach is very general, and it can be used with today's modern computational methods and equipment to compute the characteristics of complex configurations of circular plate type antenna elements. The integral equation is then solved for the unknown induced current density using numerical techniques such as the method of moment (M.O.M).

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II. Design Procedure

A circular ring of radius *a* carries a uniform charge ρ_L C/m and is placed on *X-Y* plane with axis on *Z* axis.

Using electrostatic for line charge,

$$\vec{E} = \int_L \frac{\rho_L}{4\pi\varepsilon_0 R^2} dl\, a_R \tag{1}$$

In this case,

$$dl = a d\phi$$
$$\vec{R} = a(-\vec{a}_\rho) + h\vec{a}_z \tag{2}$$

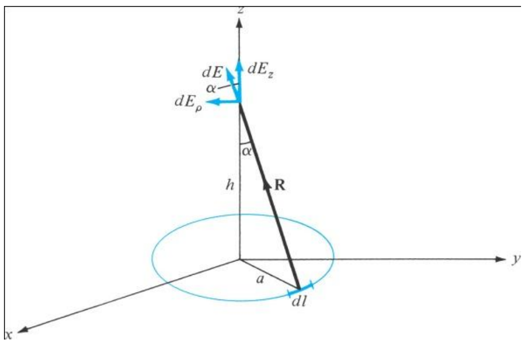

Fig. 1. Evaluation of E field due to charged ring of radius *a*.

Let, $$R = \left|\vec{R}\right| = \left[a^2 + h^2\right]^{1/2} \tag{3}$$

$$a_R = \frac{\vec{R}}{R}$$
$$\frac{a_R}{R^2} = \frac{\vec{R}}{R^3} = \frac{-a\vec{a}_\rho + h\vec{a}_z}{\left[a^2 + h^2\right]^{3/2}} \tag{4}$$

Therefore, field E will be,

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0} \int_{\phi=0}^{\phi=2\pi} \frac{(-a\vec{a}_\rho + h\vec{a}_z)}{\left[a^2 + h^2\right]^{3/2}} a d\phi \tag{5}$$

By symmetry, the contribution along a_ρ add up to zero[3]. This is evident from the fact that for every element dl, there is a corresponding element diametrically opposite that gives an equal but opposite dE_ρ so that the two contributions cancel each other. Thus we are left with the z component. That is,

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$$\vec{E} = \frac{\rho_L a h \vec{a}_z}{4\pi\varepsilon_0 \left(h^2+a^2\right)^{3/2}} \int_{\phi=0}^{\phi=2\pi} d\phi$$

$$\vec{E}_{(0,0,h)} = \frac{\rho_L a h \vec{a}_z}{2\varepsilon_0 \left(h^2+a^2\right)^{3/2}} \qquad (6)$$

For maximum value of E, h will be,

$$\frac{d\left|\vec{E}\right|}{dh} = \frac{\rho_L a}{2\varepsilon_0} \left\{ \frac{\left(h^2+a^2\right)^{3/2} - \frac{3}{2} 2h^2 \left(h^2+a^2\right)^{1/2}}{\left(h^2+a^2\right)^3} \right\} \qquad (7)$$

For Maximum E,

$$\frac{d\left|\vec{E}\right|}{dh} = 0$$

This implies that

$$\left[h^2+a^2\right]^{1/2}\left[h^2+a^2-3h^2\right]=0$$

$$a^2-2h^2=0$$

Or

$$h = \pm\frac{a}{\sqrt{2}} \qquad (8)$$

Let the charge is uniform distributed on the circular plate, the charge density is,

$$\rho_L = \frac{Q}{2\pi a}$$

So that,

$$\vec{E} = \frac{Qh}{4\pi\varepsilon_0\left[h^2+a^2\right]^{3/2}}\vec{a}_z \qquad (9)$$

As if $a \to 0$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 h^2}\vec{a}_z$$

In general,

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{a}_R \qquad (10)$$

Which is the same as that of point charge, as one would expect?

III. MATHEMATICAL FORMULATION

The target of the Integral Equation (IE) method for antenna is to cite the solution for the unknown current density[4], which is induced on the surface of the antenna, in the form of an integral equation where the unknown induced current density is part of the integrand. Potential due to a given charge distribution is considered in electrostatics, and now we know that a linear charge distribution creates an electric potential due to ring of radius a.

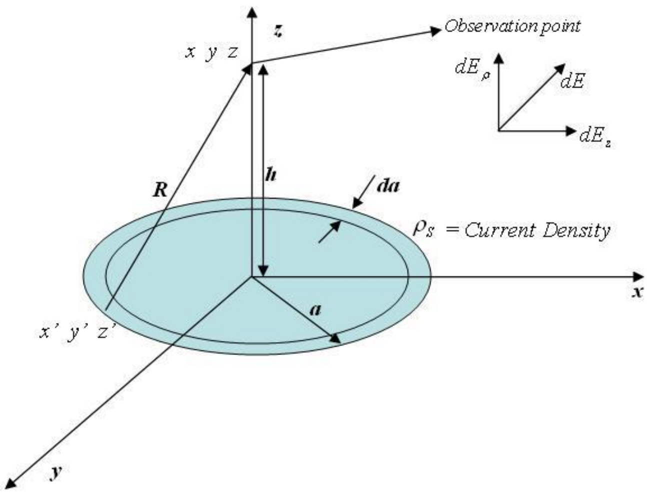

Fig. 2. Evaluation of E field due to charged circular surface of radius *a*.

Similarly for a circular disk of radius a uniformly charged with ρ_s C/m^2. The disk lies on the *z=0* plane with its axis along the *z* axis. One can calculate the E field as specified above.

$$\vec{E}_{(0,0,h)} = \frac{\rho_s}{2\varepsilon_0}\left\{1-\frac{h}{\left(h^2+a^2\right)^{1/2}}\right\}\vec{a}_z \qquad (11)$$

For far field calculation, *h*>>*a*, therefore, potential at (0,0,*h*) will be,

$$V_{(0,0,h)} = \frac{h.\rho_s}{2\varepsilon_0}\left\{1-\frac{h}{\left(h^2+a^2\right)^{1/2}}\right\} \qquad (12)$$

Let us assume the unknown charge distribution ρ_s by an expansion of *K* known terms with constant, but unknown, coefficients, that is[5],

$$\rho_s = \sum_{i=1}^{K} c_i g_i \qquad (13)$$

The circular disk of radius *a*, is divided into *k* uniform ring, each of width ζ = *a/k*. To obtain a solution for these *k* amplitude constants, *k* linearly independent equations are required. These equations may be produced by choosing *k* observation points each at the center of each ζ length element[6]. To avoid complexity in this solution, sub domain piecewise constant functions will be used. These functions, are defined to be of a constant value over one segment and zero else where, or

$$g_{i(X')} = \begin{pmatrix} 1 & & 0 < x' > a \\ & \Leftrightarrow & \\ 0 & & x' \geq a \end{pmatrix}$$

Corresponding to each observation point, For *K* such points, we can reduce 12 to

$$2\varepsilon_0 = c_1\int_0^{\zeta} \frac{g_{1(X')}}{(h^2_{1(X')}+a^2)^{3/2}}da + \dots + c_K\int_{(K-1)\zeta}^{a} \frac{g_{K(X')}}{(h^2_{1(X')}+a^2)^{3/2}}da$$

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$$2\varepsilon_0 = c_1\int_0^{\zeta} \frac{g_{1(x')}}{(h^2_{K(X')}+a^2)^{3/2}}da + \dots + c_K\int_{(K-1)\zeta}^{a} \frac{g_{K(x')}}{(h^2_{K(X')}+a^2)^{3/2}}da \quad (14)$$

We may write equation (14) more concisely using matrix notation as[7-8]

$$[V_m] = [Z_{mn}][I_n] \quad (15)$$

Where Z_{mn}, is

$$Z_{mn} = \int_0^{a} \frac{g_{K(X')}}{(h^2_{K(X')}+a^2)^{3/2}}da \quad (16)$$

If we compare eq. 15 with eq. 16,

$$[I_n] = [C_K]$$

$$[V_m] = [2\varepsilon_0]$$

The V_m column matrix has all terms equal to $2\varepsilon_0$, and the $[I_n]=[C_K]$ values are the unknown charge distribution coefficients. Solving 15 for $[I_n]$ gives

$$[I_n] = [C_K] = [Z_{mn}]^{-1}[V_m] \quad (17)$$

Equation 17 may be solved on a computer by using any of a number of matrix inversion routines[9]. The integral involved here can be evaluated in closed form by making appropriate approximations. Efficient numerical integral computer subroutines are commonly available in easy-to-use forms.

IV. Conclusion

In this paper, we have applied the M.O.M for plane circular type antenna. The integral equation for a thin circular plate is derived, some properties of integral equation are presented and are utilized to reduce the computation of integral equation to some sparse matrix notation. The method is computationally attractive, and mathematically formulation is demonstrated through illustrative example.

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TABLE I
Description for Quantity Used

Symbol	Quantity	Numerical Value
ρ	Charge density	(C/m)
θ	Elevation angle	$0 \rightarrow \pi$
ϕ	Azimuthal angle	$0 \rightarrow 2\pi$
Q	Charge C	$1.6\text{x}10^{-19}$ C
E	Electrical field	N C^{-1}
ε_0	Permittivity of Vacuum	8.85 pF m^{-1}
h	Observation point distance	In meter
a	Radius of circular plate	In meter
μ	Permeability	$1 \rightarrow 4\pi \times 10^{-7}$ H/m $= 4\pi \times 10^{-7}$ Wb/(A·m)
μ_r	Relative permeability	$\mu \rightarrow \mu_r$